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Abstract. Based on quasilinear theory, a closure scheme for anisotropic multi-component fluid equations is developed for the wave-particle interactions of ions with electromagnetic Alfvén and ion-cyclotron waves propagating along the mean magnetic field. Acceleration and heating rates are calculated. They may be used in the multi-fluid momentum and energy equations as anomalous transport terms. The corresponding evolution equation for the average wave spectrum is established, and the effective growth/damping rate for the spectrum is calculated. Given a simple power-law spectrum, an anomalous collision frequency can be derived which depends on the slope and average intensity of the spectrum, and on the gyrofrequency and the differential motion (with respect to the wave frame) of the actual ion species considered. The wave-particle interaction terms attain simple forms resembling the ones for collisional friction and temperature anisotropy relaxation (due to pitch angle scattering) with collision rates that are proportional to the gyrofrequency but diminished substantially by the relative wave energy or the fluctuation level with respect the background field. In addition, a set of quasilinear diffusion equations is derived for the reduced (with respect to the perpendicular velocity component) velocity distribution functions (VDFs), as they occur in the wave dispersion equation and the related dielectric function for parallel propagation. These reduced VDFs allow one to describe adequately the most prominent observed features, such as an ion beam and temperature anisotropy, in association with the resonant interactions of the particles with the waves on a kinetic level, yet have the advantage of being only dependent upon the parallel velocity component.

1 Introduction

It is well known that in the high-speed solar wind in interplanetary space the heavy ions move faster and have higher temperatures than the protons. For reviews of these solar wind phenomena see, e.g., Marsch (1991), concerning the early in situ measurements made by Helios, and von Steiger et al. (1995), with respect to the recent observations of Ulysses. These two space missions explored the heliosphere in the ecliptic near the Sun and out of the ecliptic. Obviously, the minor ion species can be considered as test particles, which probe the Alfvén waves and MHD turbulence in the wind and are heated and accelerated by wave-particle interactions. They are usually strongest among the waves with frequencies near the ion gyrofrequencies and those particles moving at speeds that enable them to stay in resonance with the wave electric field and to exchange thereby energy and momentum with the waves. If the relative wave amplitudes are sufficiently small, which is the case in the solar wind kinetic regime, then quasilinear theory (QLT), e.g. Davidson (1972), is adequate to describe the wave-particle coupling processes and the evolution of the particle distribution functions (VDFs), their derived moments and the wave spectra.

QLT is quadratically nonlinear in the coupling terms between the fluctuations of the velocity distribution functions and the electromagnetic fields, but linear in the sense, that these two types of fluctuations enter linearly in their product. Hence the name QLT has been coined for this weak kinetic turbulence theory, in which only the reaction of the zeroth-order VDFs on the broadband wave spectrum is considered, while the wave-wave interactions and higher-order wave-particle interactions are neglected. The wave properties (such as dispersion and growth) are evaluated from linear dispersion theory with slowly time-varying VDFs. Possible nonlinear effects in the solar wind still need to be investigated. Their potential impact has only recently been analyzed by Daughton et al. (1998) in numerical hybrid simulations of the electromagnetic proton-proton instability.

For the solar wind case, Hollweg (1974, 1978), Hollweg and Turner (1978), Revathy (1978), McKenzie et al. (1978), Dusenbery and Hollweg (1981), McKenzie and Marsch (1982), Marsch et al. (1982a), and Isenberg and Hollweg (1983) have calculated and modelled the Alfvén-wave related nonresonant and the ion-cyclotron-wave associated resonant heating and acceleration. These calculations require usually the full knowledge of the detailed wave spectrum and thus involve complicated integrals over the velocity distributions and spectral densities, which were usually assumed to be given by bi-Maxwellians and power-laws, respectively. Isenberg and Hollweg (1982) also analysed this problem from the multi-ion-fluid point of view and generalized the concept of wave-action conservation. McKenzie et al.(1993) studied more recently the subtleties of the wave dispersion and couplings in case of ion differential streaming.

We derive here the basic and related formulae of quasilinear theory in a new way with the aim to obtain closure of the multi-fluid equations with simple relaxation-time forms of the heating and acceleration rates. We will specify the quasilinear diffusion equation to the special case of wave propagation along the mean field and then develop a coupled set of diffusion equations for the two relevant reduced velocity distribution functions to be defined below. The heating and acceleration rates can be expressed in terms of these functions and transport coefficients involving wave-vector integrals over the wave power spectrum. These coefficients can be cast into a simple form for resonant interactions, involving just the spectrum at the resonant wave vector. The relevant time scale is proportional to the respective gyroperiod of any ion species considered, but is modified by the average wave fluctuation level and turns out to be of the order inferred from in situ solar wind measurements by Marsch and Richter (1987).

The main purpose of this paper is to establish the reduced diffusion equations and to develop a set of comparatively simple closure relations usable in anisotropic multi-fluid equations, in order to describe the cyclotron-resonant interactions of ions with Alfvén and ion-cyclotron or fast magnetosonic waves in the solar wind and the Sun's corona. Recent spectroscopic observations of the widths of Extreme Ultraviolet (EUV) emission lines as obtained from measurements made on the Solar and Heliospheric Observatory (SOHO) indicate that coronal heavy ions, coming in various ionization stages in the corona, are rather hot (Seely et al., 1997; Kohl et al. 1997; Wilhelm et al., 1998), particularly in the polar coronal holes where the electrons are cold, and seem to show some ordering of their kinetic temperatures according to the local gyrofrequencies (Tu et al., 1998), a result which hints to wave-particle processes as being responsible for the coronal heating.

Many years ago, Marsch et al. (1982a) and Isenberg and Hollweg (1983) have modelled alpha-particle and heavy-ion temperatures in the near-Sun solar wind at distances beyond $10R_\odot$, thereby employing the quasilinear heating rates. Recently Hu et al. (1997), Li et al. (1997) and Czechowski et al. (1998) have done anew anisotropic multi-fluid calculations, yet with adhoc mass-proportional heating functions for the heavy ions in the corona and wind. This paper will provide a more detailed physical picture of the wave heating process and give simple algebraic expressions for the heating and acceleration rates, which may be employed in the multi-fluid equations for future modelling purposes. If the detailed features of the observed VDFs are to be accounted for in kinetic models incorporating the relevant wave-particle interactions (for a Coulomb-collisional wind model of the protons see Livi and Marsch, 1987), then the reduced or full diffusion equations are certainly to be used.

2 Quasilinear theory

Quasilinear theory was developed more than twenty years ago (see e.g. Davidson, 1972) and did not evolve any further lately, because direct numerical simulations of the Maxwell equations and the particles' equations of motion have become more fashionable and convenient. This is true for homogeneous systems but much less so for inhomogeneous systems, such as the solar corona and wind, for which multi-fluid models are mainly in use, in particular if multiple species are to be modelled. We reiterate shortly the main equations of QLT. Usually, the wave fields are decomposed in plane waves with frequency ω_k and wave vector k, assumed to be directed here parallel to the background field, $\mathbf{B}_0 = B_0\mathbf{e}_x$. The Fourier component of the magnetic field is $\delta\tilde{\mathbf{B}}_k$. The spectral energy density of the magnetic field is given by $\mathcal{B}_k = \frac{1}{8\pi} \mid \delta\tilde{\mathbf{B}}_k \mid^2$ and evolves according to

$$\frac{\partial}{\partial t}\mathcal{B}_k = 2\gamma_k\mathcal{B}_k \tag{1}$$

which follows from the Fourier decomposition

$$\delta\mathbf{B}(x,t) = \int_{-\infty}^{\infty} dk\delta\tilde{\mathbf{B}}_k e^{ikx} e^{-i\int_0^t dt' z_k(t')} \tag{2}$$

where x is the coordinate along B_0, and $k > 0$ means parallel and $k < 0$ anti-parallel propagation. The growth rate, γ_k, or damping rate if it is negative, together with the real frequency, ω_k, give the complex frequency, $z_k = \omega_k + i\gamma_k$, whereby one has $\omega_k = -\omega_{-k}$, $\gamma_k = +\gamma_{-k}$, and thus $z_k^* = -z_{-k}$, and also $\delta\tilde{\mathbf{B}}_k^* = \delta\tilde{\mathbf{B}}_{-k}$, since the magnetic field in equation (2) must be real. The magnetic fluctuation energy density can be normalized to the background value such that $\hat{\mathcal{B}}_k = \frac{\mathcal{B}_k}{B_0^2/8\pi}$, with $\mathcal{B}_k = \mathcal{B}_{-k}$ by definition. Of course the distribution function $f_j(V_\perp, V_\parallel)$ is real, i.e. $f_j^* = f_j$. It is often more adequate to use the Doppler-shifted frequency, $z_k' = z_k - kU_j$, as measured in the species j frame of reference moving with its bulk speed U_j, drifting along $\mathbf{B}_0$. In this proper frame of species j, the velocities are obtained by replacing the inertial frame coordinates as follows: $V_\perp \to w_\perp$, $V_\| \to V_\| - U_j = w_\|$. A particle which moves in that frame with the resonance speed

$$w_j^\pm = \frac{z'_k \pm \Omega_j}{k} = \frac{z_k - kU_j \pm \Omega_j}{k} \tag{3}$$

sees a stationary electric field and, thus being in cyclotron resonance, does very effectively exchange energy and momentum with the wave. The diffusion equation describes the evolution of the velocity distribution function in the inertial frame of reference, in which the particles and waves are supposed to propagate, and is after Davidson (1972) and Marsch et al. (1982a) given as

$$\frac{\partial}{\partial t} f_j(w_\perp, w_\|, t) = \Omega_j^2 \frac{1}{4} \sum_{+,-} \int_{-\infty}^{\infty} dk \mathcal{B}_k$$
$$\times \Im \left[w_\perp \frac{\partial}{\partial w_\|} - \left(\frac{z'_{-k}}{k} + w_\| \right) \frac{1}{w_\perp} \frac{\partial}{\partial w_\perp} w_\perp \right]$$
$$\times \frac{1}{k(w_\| - w_j^\pm(k))} \left[w_\perp \frac{\partial}{\partial w_\|} + \left(\frac{z'_k}{k} - w_\| \right) \frac{\partial}{\partial w_\perp} \right] f_j \tag{4}$$

Remember that the frequency z_k might be complex for strong damping or growth, and thus the imaginary part, indicated by the symbol $\Im$ in front of the brackets, refers to the resonant denominator as well as to the complex frequencies in the differential operators. Since in a multi-component plasma each species contributes its own wave mode (see e.g. Mann et al., 1997), we may tacitly assume that the sum in (4) includes a summation over the various dispersion branches, whereby the random-phase approximation ensures that no constructive interference occurs between different modes. We omit the summation index here for the sake of lucidity.

3 Heating and acceleration rates

We take velocity moments of $\frac{\partial}{\partial t} f_j$ as given in equation (4). The zeroth moment is

$$< \frac{\partial}{\partial t} f_j >= 2\pi \int_0^\infty dw_\perp w_\perp \int_{-\infty}^{\infty} dw_\| \frac{\partial}{\partial t} f_j = 0 \tag{5}$$

expressing conservation of n_j. Note that the distribution function vanishes at infinity, which implies that $f_j(w_\perp, \pm\infty) = 0$ and $f_j(\infty, w_\|) = 0$. The first moment gives the bulk acceleration:

$$\frac{\partial}{\partial t} U_j =< w_\| \frac{\partial}{\partial t} f_j > \tag{6}$$

The heating rates are defined by the second moments and given as

$$\frac{\partial}{\partial t} V_{j\|}^2 =< w_\|^2 \frac{\partial f_j}{\partial t} > \tag{7}$$

$$\frac{\partial}{\partial t} V_{j\perp}^2 =< \frac{w_\perp^2}{2} \frac{\partial f_j}{\partial t} > \tag{8}$$

By summing up equations (6,7,8) we obtain the total rate of change of the thermal and kinetic energy for the particles of species j. This relation can be further summed up over all particle species and, by using the dispersion relation, further modified to obtain the total energy conservation law within QLT for a multi-component plasma (see Davidson, 1972). By taking the limit $(V_A/c)^2 \to 0$ the total energy conservation law may be written in the form:

$$\frac{\partial}{\partial t} \left\{ \sum_j \frac{1}{2} \rho_j (V_{j\|}^2 + 2V_{j\perp}^2 + U_j^2) + \int_{-\infty}^{\infty} dk \frac{\delta \tilde{\mathbf{B}}_k^2}{8\pi} \right\} = 0 \tag{9}$$

Note that this relation does not depend on the details of the VDF or the wave spectrum or the dispersion characteristics, a property which shall be used later on to impose energy conservation on the closure relations to be derived. The mean thermal speed parallel and perpendicular to the field are defined by the second moments, $V_{j\|}^2 =< w_\|^2 >_j$ and $V_{j\perp}^2 =< \frac{w_\perp^2}{2} >_j$. Here the brackets stand for the full velocity space integration and the index j refers to the respective VDF.

4 Dispersion relation and dielectric function

The dispersion equation for parallel propagating left (− sign) and right (+ sign) handed circularly-polarized electromagnetic waves reads, e.g. after Dum et al. (1980), as follows:

$$\left(\frac{ck}{z_k} \right)^2 = 1 + \sum_j \left(\frac{\omega_j}{z_k} \right)^2 \hat{\varepsilon}_j^\pm (z'_k, k) \tag{10}$$

with the speed of light denoted by c. The dielectric constant involves the resonance integral over the distribution function and reads

$$\hat{\varepsilon}_j^\pm = 2\pi \int_0^\infty dw_\perp w_\perp \int_{-\infty}^{\infty} dw_\| \frac{w_\perp/2}{w_\| - w_j^\pm}$$
$$\times \left((w_\| - \frac{z'_k}{k}) \frac{\partial}{\partial w_\perp} - w_\perp \frac{\partial}{\partial w_\|} \right) f_j(w_\perp, w_\|, t) \tag{11}$$

To evaluate this function only the knowledge of two reduced distribution functions (after Dum et al., 1980) is required. They are discussed in the next section. The distribution function f_j is understood to be normalized to the number density n_j. The following definitions hold: The ion charge is e_j, the mass is m_j, and the plasma frequency is $\omega_j^2 = \frac{4\pi e_j^2 n_j}{m_j}$. The ion gyrofrequency, given by the definition carrying the sign of the charge, reads as follows $\Omega_j = \frac{e_j B_0}{m_j c}$. Note that the dispersion equation (10) can also be rewritten as

$$\left(\frac{kV_A}{z_k}\right)^2 = \left(\frac{V_A}{c}\right)^2 + \sum_j \hat{\rho}_j \left(\frac{\Omega_j}{z_k}\right)^2 \hat{\varepsilon}_j^{\pm}(z'_k, k) \qquad (12)$$

where the fractional mass density of species j has been used, which is defined as $\hat{\rho}_j = n_j m_j / \sum_\ell n_\ell m_\ell$, and the Alfvén velocity based on the total mass density, $V_A^2 = B_0^2/(4\pi\rho)$, has been used to normalize the phase speed properly.

5 Diffusion equation for the reduced distribution functions

The evaluation of the dispersion equation and the heating and acceleration rates do not involve the full two-dimensional VDF but only the two reduced velocity distributions defined as follows:

$$F_{j\|}(w_\|) = 2\pi \int_0^\infty dw_\perp w_\perp f_j(w_\perp, w_\|) \qquad (13)$$

$$F_{j\perp}(w_\|) = 2\pi \int_0^\infty dw_\perp w_\perp \frac{w_\perp^2}{2V_A^2} f_j(w_\perp, w_\|) \qquad (14)$$

In terms of these VDF the dielectric function (11) can be written as

$$\begin{aligned}\hat{\varepsilon}_j^{\pm} \;&=\; \int_{-\infty}^{\infty} dw_\| \frac{1}{w_j^{\pm} - w_\|} \left((w_\| - \frac{z'_k}{k}) F_{j\|}(w_\|) \right. \\ &+\; \left. V_A^2 \frac{\partial}{\partial w_\|} F_{j\perp}(w_\|) \right) \qquad (15)\end{aligned}$$

Note that the first is a genuine particle VDF and the second corresponds to the distribution of the perpendicular plasma beta in dependence upon the parallel speed. The temporal rates of change of the reduced distribution functions are calculated by taking the corresponding moments of the diffusion equation (4) with the result:

$$\frac{\partial}{\partial t} F_{j\|} = 2\pi \int_0^\infty dw_\perp w_\perp \frac{\partial}{\partial t} f_j(w_\perp, w_\|, t) \qquad (16)$$

$$\frac{\partial}{\partial t} F_{j\perp} = 2\pi \int_0^\infty dw_\perp w_\perp \frac{w_\perp^2}{2V_A^2} \frac{\partial}{\partial t} f_j(w_\perp, w_\|, t) \qquad (17)$$

By partial integration the evolution equation for the parallel reduced VDF defined in equation (13) reads

$$\begin{aligned}&\frac{\partial}{\partial t} F_{j\|}(w_\|) \\ =\;& \Omega_j^2 \frac{1}{2} \sum_{+,-} \int_{-\infty}^{\infty} dk \hat{\mathcal{B}}_k \frac{\partial}{\partial w_\|} \Im \left\{ \frac{1}{k(w_\| - w_j^{\pm}(k))} \right. \\ &\times \left. \left(\frac{\partial}{\partial w_\|} (V_A^2 F_{j\perp}) - \left(\frac{z'_k}{k} - w_\| \right) F_{j\|} \right) \right\} \qquad (18)\end{aligned}$$

It depends also on the reduced perpendicular VDF defined in equation (14) and thus the corresponding diffusion equation is also required. It is quoted below. It is meaningful to define the "transport" functions for diffusion and acceleration, respectively deceleration, by the wave-vector integrals:

$$D_j(w_\|) = \Omega_j^2 \frac{1}{2} \sum_{+,-} \int_{-\infty}^{\infty} dk \hat{\mathcal{B}}_k \Im \left\{ \frac{V_A^2}{k(w_\| - w_j^{\pm}(k))} \right\} \qquad (19)$$

$$A_j^{+}(w_\|) = \Omega_j^2 \frac{1}{2} \sum_{+,-} \int_{-\infty}^{\infty} dk \hat{\mathcal{B}}_k \Im \left\{ \frac{z'_k/k - w_\|}{k(w_\| - w_j^{\pm}(k))} \right\} \qquad (20)$$

These diffusion coefficients and acceleration rates, which of course still depend on the parallel speed, $w_\|$, through the resonance denominator, describe in a transparent way the diffusive broadening of the VDF and the acceleration of a particle, which ceases if the particle co-moves in the wave frame of reference with the wave having a phase velocity z'_k/k. The net effect of these processes is obtained by integration over the entire wave spectrum. Note that the imaginary part of the denominator involves nonresonant as well as resonant particles which travel at the speed $w_j^{\pm}(k)$. If γ_k becomes vanishingly small, then the imaginary part becomes a delta function and thus the resonance gets sharp. For the parallel VDF we obtain a transport equation in the form

$$\begin{aligned}\frac{\partial}{\partial t} F_{j\|}(w_\|) \;&=\; \frac{\partial}{\partial w_\|} D_j(w_\|) \frac{\partial}{\partial w_\|} F_{j\perp}(w_\|) \\ &-\; \frac{\partial}{\partial w_\|} \left(A_j^{+}(w_\|) F_{j\|}(w_\|) \right) \qquad (21)\end{aligned}$$

For the perpendicular VDF we obtain a transport equation in the form

$$\begin{aligned}&\frac{\partial}{\partial t} F_{j\perp}(w_\|) = \frac{\partial}{\partial w_\|} D_j(w_\|) \frac{\partial}{\partial w_\|} F_{j\perp}^{(2)}(w_\|) \\ -\;& 2 \frac{\partial}{\partial w_\|} \left(A_j^{+}(w_\|) F_{j\perp}(w_\|) \right) \\ +\;& A_j^{-}(w_\|) \frac{\partial}{\partial w_\|} F_{j\perp}(w_\|) - H_j(w_\|) F_{j\|}(w_\|) \qquad (22)\end{aligned}$$

which involves two more coefficients, having the dimension of an acceleration and temporal rate of change. These are again functions of $w_\|$ and read

$$A_j^{-}(w_\|) = \Omega_j^2 \frac{1}{2} \sum_{+,-} \int_{-\infty}^{\infty} dk \hat{\mathcal{B}}_k \Im \left\{ \frac{z'_{-k}/k + w_\|}{k(w_\| - w_j^{\pm}(k))} \right\} \qquad (23)$$

$$\begin{aligned}H_j(w_\|) \;&=\; \Omega_j^2 \frac{1}{2} \sum_{+,-} \int_{-\infty}^{\infty} dk \hat{\mathcal{B}}_k \Im \left\{ \frac{1/V_A^2}{k(w_\| - w_j^{\pm}(k))} \right. \\ &\times \left. \left(\frac{z'_k}{k} - w_\| \right) \left(\frac{z'_{-k}}{k} + w_\| \right) \right\} \qquad (24)\end{aligned}$$

In (22) we were forced to introduce another higher-order reduced VDF given by

$$F_{j\perp}^{(2)}(w_{\|}) = 2\pi \int_0^{\infty} dw_{\perp} w_{\perp} \frac{w_{\perp}^4}{4V_A^4} f_j(w_{\perp}, w_{\|}) \tag{25}$$

for which in principle another evolution equation must be derived to obtain its time evolution. Apparently, we are facing the difficult problem of closure for the reduced VDFs. An infinite chain of evolution equations would result if no approximation would be made. Therefore, the advantage of using 1-D VDFs instead of the 2-D original one would be lost. To break the chain of higher-order moments we therefore make the Gaussian approximation

$$F_{j\perp}^{(2)}(w_{\|}) = 2\beta_{j\perp} F_{j\perp}(w_{\|}) \tag{26}$$

which would be exact for a bi-Maxwellian. Of course, this does not imply that $F_{j\|,\perp}$ is Gaussian. With the relation (26) being inserted, the equations (21, 22) now form a closed set of diffusion equations, which can be solved given the transport coefficients, i.e. the wave spectrum, is known. The price to be payed for closure is that the evolution equation for $F_{j\perp}$ is now an integro-differential equation, since the plasma beta is defined by the first parallel moment of $F_{j\perp}$. The dependence on U_j is not essential, since it can be removed by going into the plasma (center-of-mass) frame. The two reduced VDFs allow one to describe still adequately the most prominent and relevant observed features, such as an ion beam and variable temperature anisotropy along the field (Marsch et al., 1982b,c) in association with the resonant interactions of the particles with the waves on a kinetic level, yet they have the advantage of being only dependent upon the parallel velocity component. Their linked evolution equations (21, 22) are less complex than the original two-dimensional diffusion equation (4). This result is particularly advantageous if one seeks numerical solutions. Note that by definition the following normalizations hold:

$$\int_{-\infty}^{\infty} dw_{\|} F_{j\|}(w_{\|}) = 1 \tag{27}$$

The definition of the bulk speed implies that the first moment

$$\int_{-\infty}^{\infty} dw_{\|} w_{\|} F_{j\|}(w_{\|}) = 0 \tag{28}$$

vanishes. The squared parallel thermal speed is calculated as the second moment of the reduced distribution

$$\int_{-\infty}^{\infty} dw_{\|} w_{\|}^2 F_{j\|}(w_{\|}) = V_{j\|}^2 \tag{29}$$

Because of the definition (14) the constraint on $F_{j\|}$,

$$\int_{-\infty}^{\infty} dw_{\|} F_{j\perp}(w_{\|}) = \frac{V_{j\perp}^2}{V_A^2} = \beta_{j\perp} \tag{30}$$

is obtained. Higher-order moments, such as heat fluxes, will not be considered here.

6 Heating and acceleration rates as reduced velocity moments

With the preparations of the previous sections, it is now straightforward to evaluate the rates of change of the thermal speeds (or temperatures) and of the bulk speed of species j by taking the first three parallel velocity moments of equations (21, 22), with the results:

$$\begin{aligned} \frac{\partial}{\partial t} U_j = < w_{\|} \frac{\partial F_{j\|}}{\partial t} >_{\|} &= - < D_j \frac{\partial}{\partial w_{\|}} F_{j\perp} >_{\|} \\ &+ < A_j^{+} F_{j\|} >_{\|} \end{aligned} \tag{31}$$

$$\begin{aligned} \frac{\partial}{\partial t} V_{j\|}^2 = < w_{\|}^2 \frac{\partial F_{j\|}}{\partial t} >_{\|} &= -2 < w_{\|} D_j \frac{\partial}{\partial w_{\|}} F_{j\perp} >_{\|} \\ &+ 2 < w_{\|} A_j^{+} F_{j\|} >_{\|} \end{aligned} \tag{32}$$

$$\begin{aligned} \frac{\partial}{\partial t} V_{j\perp}^2 = < V_A^2 \frac{\partial F_{j\perp}}{\partial t} >_{\|} &= V_A^2 \Big(< A_j^{-} \frac{\partial}{\partial w_{\|}} F_{j\perp} >_{\|} \\ &- < H_j F_{j\|} >_{\|} \Big) \end{aligned} \tag{33}$$

Here the brackets with index $\|$ refer to an integration over $w_{\|}$ only. These rates are mathematically entirely equivalent to the rates based on the dielectric functions as given in Marsch et al. (1982a). However, note the different emphasis here, where the character of the velocity moments is retained and the waves appear only through the transport coefficients. In contrast, the former version stresses the waves and indicates that the particle heating and acceleration is obtained by an integration over the wave spectrum, whereby the imaginary parts of the dielectric functions represent a kind of wave-energy absorption coefficients and depend sensitively on the shape of the VDFs.

7 General transport coefficients

For non-dispersive or Alfvén waves only the diffusion coefficient is essentially needed, and the other coefficients can be expressed by it. In the general dispersive case with a finite γ_k, i.e. for dissipative and nonresonant wave-particle interactions, we can also formally write the transport coefficients in a way which is physically transparent and gives immediately the Alfvén-wave limit. For this purpose let us define a weighted average over the full spectrum defined as follows:

$$\ll f(k) \gg_j = \frac{\sum_{+,-} \int_{-\infty}^{\infty} dk \hat{\mathcal{B}}_k \Im \left\{ \frac{f(k)}{k(w_{\|} - w_j^{\pm}(k))} \right\}}{\sum_{+,-} \int_{-\infty}^{\infty} dk \hat{\mathcal{B}}_k \Im \left\{ \frac{1}{k(w_{\|} - w_j^{\pm}(k))} \right\}} \tag{34}$$

which is defined for any complex-valued function $f(k)$ and normalized to unity for $f(k) = 1$. The general relaxation time can be defined as

$$\frac{1}{\tau_j(w_{||})} = \Omega_j^2 \frac{1}{2} \sum_{+,-} \int_{-\infty}^{\infty} dk \hat{\mathcal{B}}_k \Im \left\{ \frac{1}{k(w_{||} - w_j^{\pm}(k))} \right\} \quad (35)$$

With these definitions we can write the transport coefficients as

$$D_j(w_{||}) = \frac{V_A^2}{\tau_j(w_{||})} \quad (36)$$

$$A_j^{\pm}(w_{||}) = \frac{1}{\tau_j(w_{||})} \left(\ll \frac{z_{\pm k}}{k} \gg_j \mp (w_{||} + U_j) \right) \quad (37)$$

$$H_j(w_{||}) = -\frac{1}{\tau_j(w_{||})} \frac{1}{V_A^2} \left(\ll \frac{|z_k|^2}{k^2} \gg_j - 2(w_{||} + U_j) \ll \frac{\Re z_k}{k} \gg_j + (w_{||} + U_j)^2 \right) \quad (38)$$

These last three equations have a particularly simple form, in which everything is reduced to the associated time scale (35). The case of resonant wave-particle interactions corresponds to weak growth or small damping, i.e. to the limit $\gamma_k \to 0$. The functions $D_j, A_j^{\pm}$ and H_j all depend on the imaginary part

$$\Im \left\{ \frac{1}{k(w_{||} - w_j^{\pm})} \right\} = \frac{\gamma_k}{(\omega_k \pm \Omega_j - k w_{||} - k U_j)^2 + \gamma_k^2} \quad (39)$$

which can be re-written in the resonant limit by help of the delta function properties. This limit reproduces the well-known form,

$$D_j(w_{||}) = \Omega_j^2 V_A^2 \frac{\pi}{2} \times \sum_{+,-} \int_{-\infty}^{\infty} dk \hat{\mathcal{B}}_k \delta(\omega_k \pm \Omega_j - k w_{||} - k U_j)) \quad (40)$$

of the diffusion coefficient. Similar expressions can be derived for the other two transport coefficients.

8 Closure relations for anisotropic multi-fluid equations

8.1 Phase velocity in a multi-ion plasma

We derive the phase velocity of low-frequency waves propagating along the magnetic field in a multi-ion plasma. This velocity can also be obtained directly from the multi-fluid equations (Isenberg and Hollweg, 1982, 1983). For a recent derivation within the context of Alfvén-wave minor-ion interactions see the paper of Mc Kenzie (1994). The warm plasma dispersion relation for a multi-component plasma was also discussed in Dum et al. (1980). The dispersion relation for a multi-ion plasma was recently discussed extensively by Mann et al. (1997). We start below from our general dispersion relation (10). The large-resonant-speed expansions of the normalized dielectric constants, in the limit $\gamma_k \to 0$ in which $z_k' \to \omega_k'$ holds, yield the result

$$\left(\frac{kc}{\omega_k}\right)^2 = 1 + \sum_j \left(\frac{\omega_j}{\omega_k}\right)^2 \left\{ -\frac{\omega_k'}{\omega_k' \pm \Omega_j} + \frac{(V_{j||}^2 - V_{j\perp}^2)k^2}{(\omega_k' \pm \Omega_j)^2} \right\} \quad (41)$$

This can be simplified by exploiting that the background plasma bears no current and is quasineutral. It is convenient to define the center of mass velocity, $U_m = \sum_j \hat{\rho}_j U_j$, and the differential speed, $\Delta U_j = U_j - U_m$, with which each species moves relative to the center of mass frame. Remember that $\hat{\rho}_j = \rho_j/\rho$ is the fractional mass density of species j. The drift kinetic motion enters in the same way as the parallel thermal speed, and one obtains a quadratic equation for the phase speed:

$$\left(\frac{\omega_k}{kV_A}\right)^2 - 2\left(\frac{\omega_k}{kV_A}\right)\frac{U_m}{V_A} = 1 - \sum_j \hat{\rho}_j \frac{V_{j||}^2 - V_{j\perp}^2 + \Delta U_j^2}{V_A^2} - \left(\frac{U_m}{V_A}\right)^2 \quad (42)$$

which has the solution

$$\frac{\omega_k}{k} = U_m \pm \left\{ V_A^2 - \sum_j \hat{\rho}_j \left(V_{j||}^2 - V_{j\perp}^2 + \Delta U_j^2 \right) \right\}^{1/2} \quad (43)$$

This is the phase velocity of an Alfvén wave in a differentially drifting multi-ion plasma in the inertial frame. The factor in the brackets is the generalized firehose correction.

8.2 Transport coefficients for Alfvén waves

We are now in the position to write the transport coefficients for non-dispersive waves in a simple way. The phase velocity (43) can be inserted in the equations (36, 37, 38). Since it does not depend on k the averaging procedure $\ll ... \gg_j$ is trivial. The so simplified transport coefficients for the resonant case are given as

$$D_j(w_{||}) = \frac{V_A^2}{\tau_j(w_{||})} \quad (44)$$

for the diffusion coefficient, which also defines the resonant relaxation time according to (40), and

$$A_j^{\pm}(w_{||}) = \pm \frac{1}{\tau_j(w_{||})} \left(\frac{\omega_k}{k} - U_j - w_{||}\right) \quad (45)$$

for the de/acceleration or wave-induced friction and finally

$$H_j(w_{||}) = -\frac{1}{\tau_j(w_{||})} \frac{1}{V_A^2}(\frac{\omega_k}{k} - U_j - w_{||})^2 \quad (46)$$

for the specific heating. The phase speed, $V_p' = V_p - U_m = \frac{\omega}{k} - U_m$ in the center of mass frame, is assumed to be much larger than the speed $w_{||}$ in the species' frame of reference, which is consistent with $(V_{j||,\perp}/V_A)^2 = \beta_{j||,\perp} \ll 1$ and was also used in the expansion of the dielectric function. Therefore, we may use $\tau_j(0)$ to evaluate the transport coefficients in lowest order of $\beta_{j||,\perp}$, in which case the remaining parallel velocity integration can be easily performed. To discriminate this rate against the more general $w_{||}$-dependent rate, we introduce the anomalous wave-particle collision frequency, $\nu_j = 1/\tau_j(0)$.

8.3 Closure relations for the heating and acceleration rates

The special rate equations for particles interacting with Alfvén waves are obtained by insertion of (44, 45, 46) in the general rate equations (31, 32, 33) and by performing the corresponding moment integrations. We obtain

$$\frac{\partial}{\partial t}U_j = -\nu_j(U_j - V_p) \quad (47)$$

$$\frac{1}{2}\frac{\partial}{\partial t}V_{j||}^2 = -\nu_j(V_{j||}^2 - V_{j\perp}^2) \quad (48)$$

$$\frac{\partial}{\partial t}V_{j\perp}^2 = -\nu_j\left(V_{j\perp}^2 - V_{j||}^2 - (U_j - V_p)^2\right) \quad (49)$$

This result is intuitively very satisfying and comparatively simple, yet seems to retain essential physics of the resonant interaction of ions with Alfvén waves in a multicomponent plasma. Inspection of these equations shows that the waves exert a friction force, which ceases if the particles move at the phase speed of the wave, in which case the electric field of the wave has been transformed away such that no de/acceleration can occur. The parallel heating rate looks like a simple pitch angle scattering term acting on the temperature relaxation time scale τ_j. The perpendicular heating rate contains a similar anisotropy-relaxation term but more importantly a wave heating term, which again goes to zero if the particles move in the wave frame. Note that the heating is always positive, whereas the temperature relaxation term has a sign which depends on the anisotropy.

Trivially, the state where all particles have isotropic pressures in the wave frame of reference is a stable equilibrium. Ions surfing the waves are not heated any more and kept by wave friction at the phase speed. It should be stressed here, that such a state has been observed with the protons and alpha particles in the solar wind (Marsch et al., 1982b,c) and with the heavier ions as well (von Steiger et al., 1995). The isotropic heating rate is

$$Q_j = \rho_j \frac{1}{2}\frac{\partial}{\partial t}(V_{j||}^2 + 2V_{j\perp}^2) = \nu_j\rho_j(V_p - U_j)^2 \quad (50)$$

This means that the acceleration $a_j = \frac{\partial}{\partial t}U_j$ is related to the volumetric heating Q_j rate by the simple equation, $a_j(V_p - U_j) = Q_j/\rho_j$, a relation obtained before by Isenberg and Hollweg (1982, 1983), which holds for any nondispersive wave and also follows directly from equations (31, 32, 33). From the equations (48, 49) it is clear that the specific heating rate, $\frac{\partial T_j}{\partial t}$, is proportional to the mass m_j and thus favours the heavier ions. Given all species have the same speed, and given a power-law spectrum as below in equation (53) holds, then the ratio of the accelerations for species i and j is

$$\frac{a_i}{a_j} = \frac{\nu_i}{\nu_j} = \left|\frac{\Omega_i}{\Omega_j}\right|^{(2-\alpha)} \quad (51)$$

which favours preferential acceleration of those minor ions that have the largest Z_j/A_j ratio. The physics resulting from this scaling behaviour and the consequences for the interplanetary solar wind have been discussed by Isenberg and Hollweg (1983). The consequences for the nascent solar wind in coronal holes still need to be worked out.

Finally, we need to address the time change of the magnetic field fluctuation energy. Since we deal with the limit $\gamma_k \to 0$ here, we have no evolution equation for the wave spectrum. On the other hand we can exploit the overall energy conservation equation (9) and the equations (47 – 49) to calculate the rate of change in time of the integrated spectrum as

$$\begin{aligned}\frac{\partial}{\partial t}\frac{<\delta B^2>}{8\pi} &= -\sum_j \nu_j\rho_j V_p(V_p - U_j) \\ &= -V_p\sum_j \rho_j\frac{\partial}{\partial t}U_j \quad (52)\end{aligned}$$

Apparently, all species contribute here to the energy change in the magnetic field fluctuations, and particles which move faster than V_p give energy to whereas those moving slower take energy from the waves. Note that the brackets used above mean an integration over that region of k-space, i.e. $<\delta B^2> = \int_{k_L}^{k_U} dk\, \delta\tilde{\mathbf{B}}_k^2$, in which significant wave-particle intercations take place. Without information on the spectral shape, the last equation cannot be evaluated any further. In the next section we shall assume that the spectrum obeys a simple invariant power law.

8.4 Relaxation time for a power-law spectrum

In interplanetary space magnetic field fluctuations often obey simple power laws (for a review see, e.g., Tu and Marsch, 1995) with a spectral index, α, which may range observationally between 1 and 2. Let us assume, therefore, that the spectrum obeys

$$\hat{\mathcal{B}}_k = \hat{\mathcal{B}}_{k_0}\left(\frac{k}{k_0}\right)^{-\alpha} \tag{53}$$

with some free reference wave vector k_0. In the resonant limit and for nondispersive waves, the fundamental time scale, τ_j, depends upon the combination

$$\begin{aligned} \hat{\mathcal{B}}_{k_j} \mid k_j \mid &= \hat{\mathcal{B}}_{k_0} k_0 \left(\frac{k_j}{k_0}\right)^{(1-\alpha)} \\ &= \hat{\mathcal{B}}_{k_0} k_0 \left|\frac{\Omega_j}{k_0(V_p - U_j - w_{\|})}\right|^{(1-\alpha)} \end{aligned} \tag{54}$$

which also gives implicitly the definition of the resonance wave-vector k_j. We recall that $\hat{\mathcal{B}}_{k_0} k_0 \sim < (\delta B/B_0)^2 >$, and thus scales with the average relative fluctuation amplitude in the dissipation domain. Here k_0 is a free reference wave vector for k, for which the proton inertial length, $k_0 = \Omega_p/V_A$, is a good choice. Then we obtain the time scale

$$\frac{1}{\tau_j(w_{\|})} = \pi\Omega_p \hat{\mathcal{B}}_{k_0} k_0 \left|\frac{\Omega_j}{\Omega_p}\right|^{(2-\alpha)} \left|\frac{V_A}{V_p - U_j - w_{\|}}\right|^{(1-\alpha)} \tag{55}$$

Note that this time scale implies a preferential acceleration and heating of the various species according to their charge/mass ratio, i.e. with $(Z_j/A_j)^{(2-\alpha)}$, resulting from the gyrofrequency ratio. If $\alpha = 1$, then the rate is directly proportional to Z_j/A_j, otherwise the spectral slope is decisive for the differential effects, as discussed in some length in the solar wind model of Isenberg and Hollweg (1983), who derived similar scaling relations and found in their wind models that slopes with $\alpha > 2$ are required in order to accelerate and heat heavy ions such as iron ions preferentially by non-dispersive waves. The recent coronal hole observations from SOHO (Tu et al., 1998; Seely et al., 1997) seem to indicate a scaling of Ω_j with α which corresponds to α ranging between 1.5 and 2. This is similar to the spectral exponents measured in situ in the inner heliosphere (Tu and Marsch, 1995).

From equation (53) we can, by an integration between some upper limit, k_U, and lower limit, k_L, obtain the relation

$$\begin{aligned} < (\frac{\delta B}{B_0})^2 > &= \hat{\mathcal{B}}_{k_0} k_0 \frac{1}{1-\alpha}\left((\frac{k_U}{k_0})^{(1-\alpha)} \right. \\ &- \left. (\frac{k_L}{k_0})^{(1-\alpha)} \right) = \hat{\mathcal{B}}_{k_0} k_0 S_\alpha \end{aligned} \tag{56}$$

For $\alpha = 1$, the factor S_α on the right hand side is equal to $\ln(k_U/k_L)$ and gives the number of dyades contained in the spectrum. Otherwise, S_α is a sensitive function of the slope α. Apparently, a k^{-1} spectrum has the same energy content in each logarithmic spectral segment. We can now define the collision frequency associated with the wave-particle interactions as follows:

$$\nu_j = \pi\Omega_p < (\frac{\delta B}{B_0})^2 > \frac{1}{S_\alpha}\left|\frac{\Omega_j}{\Omega_p}\right|^{(2-\alpha)} \left|\frac{V_p - U_j}{V_A}\right|^{(\alpha-1)} \tag{57}$$

The strength of this anomalous collision frequency depends on the relative drift and the charge-per-mass ratio of the species j, and most importantly, on the average wave amplitude. If the spectral slope is steeper than 1, then ν_j approches zero, if it is flatter than 1, then ν_j goes to infinity, while the particles' bulk drift speed, U_j, along the mean field approaches the wave phase speed. Of course, this singularity is somewhat artificial and will be removed if a finite $w_{\|}$ is considered, i.e. if finite β_j effects are accounted for in the relaxation rate.

8.5 Closure relation for the average wave amplitude

We can now use the anomalous collision rate ν_j on the right-hand side of equation (52), in order to derive an effective damping or growth rate for the average wave energy density. For this purpose it is convenient to replace $B_0^2/(8\pi)$ by $1/2\rho V_A^2$ in the last equation. After some algebra we obtain

$$\gamma_B = \pi\Omega_p \frac{1}{S_\alpha}\frac{V_p}{V_A}\sum_j \left(\hat{\rho}_j \frac{U_j - V_p}{V_A}\left|\frac{\Omega_j}{\Omega_p}\right|^{(2-\alpha)} \times \left|\frac{V_p - U_j}{V_A}\right|^{(\alpha-1)} \right) \tag{58}$$

The wave energy develops according to the evolution equation

$$\frac{\partial}{\partial t} < (\delta B)^2 > = 2\gamma_B < (\delta B)^2 > \tag{59}$$

which is constructed similarly to the original equation (1) for the relative spectral density at a given k. This last equation is the required closure relation. Note that γ_B may have either sign. If all species lag behind the waves then γ_B is negative and the wave energy is damped. If they all run faster than the wave, then γ_B is positive, leading to wave growth and particle deceleration. If all particles surf the waves, then γ_B would formally be zero. Yet, not all particles together can surf the waves, unless the plasma frame phase speed, V_p', is zero, which is trivial. Note that γ_B differs from the rate ν_j by a factor of the order of the average fluctuation level. If this is small, the wave field changes much more rapidly than the particles' drift and thermal velocities and the wave energy will be readily depleted, resulting in weak and slow effects on the particles.

9 Approximate rates for dispersive and dissipative waves

A refined closure scheme is obtained for dispersive and dissipative waves if we make the assumption that the $w_{||}$-dependence in the phase velocity average according to (34) and the relaxation time (35) can be neglected. This approach gives further insight into the wave-particle interactions. Let us define the average speeds (that may be taken at $w_{||} = 0$ or any other typical speed for the bulk of the species j, like their thermal speed) as follows: $\mathcal{V}_j = \ll \frac{\omega_k}{k} \gg_j$, $\mathcal{W}_j = \ll \frac{i\gamma_k}{k} \gg_j$, and $\Delta\mathcal{V}_j^2 = \ll \frac{|z_k|^2}{k^2} \gg_j - (\ll \frac{\omega_k}{k} \gg_j)^2$, which is positive. Note that the averaging implies taking the imaginary part of the integrand, and thus the imaginary unit i cannot be taken out in front of the brackets. With these definitions we can write the rates as

$$\frac{\partial}{\partial t} U_j = -\frac{1}{\tau_j(0)} (U_j - \mathcal{V}_j - \mathcal{W}_j) \tag{60}$$

$$\frac{1}{2}\frac{\partial}{\partial t} V_{j||}^2 = -\frac{1}{\tau_j(0)} (V_{j||}^2 - V_{j\perp}^2) \tag{61}$$

$$\frac{\partial}{\partial t} V_{j\perp}^2 = -\frac{1}{\tau_j(0)} \left(V_{j\perp}^2 - V_{j||}^2 - \Delta\mathcal{V}_j^2 - (U_j - \mathcal{V}_j)^2 \right) \tag{62}$$

According to these equations the waves will force the particles to move at their average phase velocity, which has a dissipative and dispersive component. The parallel thermal heating rate remains unchanged. Let us assume that the particles "surf" the waves while moving at their average phase speed $\mathcal{V}_j + \mathcal{W}_j$, and that they have an isotropic temperature. The corresponding heating rate can then, after some algebra, be written as:

$$\begin{aligned} \frac{\partial}{\partial t} V_{j\perp}^2 &= \frac{1}{\tau_j(0)} \left(\ll (\frac{\omega_k}{k} - \ll \frac{\omega_k}{k} \gg_j)^2 \gg_j \right. \\ &+ \left. \ll | \frac{i\gamma_k}{k} - \ll \frac{i\gamma_k}{k} \gg_j |^2 \gg_j \right) \end{aligned} \tag{63}$$

This result gives a clear intuitive picture of the remaining isotropic heating of the particles, which is due to the variances in the real and imaginary parts of the complex phase speed. For non-dispersive and undamped waves these variances vanish, otherwise they will remain finite and the heating ceases only if the wave spectrum is decimated, in which case the rate $1/\tau_j(0)$ tends to zero. As a result, particles surfing at their mean phase speed on a broad-band dispersive wave field will always be heated.

10 Summary and conclusions

We have described in this article the interactions between ions and electromagnetic waves propagating along the mean magnetic field. A closure scheme has been established for the heating and acceleration rates of the ions in association with cyclotron-resonant or nonresonant interactions with the waves. In the low-β_j situation, which prevails in the solar corona and near-Sun solar wind, the general transport coefficients may be simplified by taking the relaxation time and the averaged phase velocity at the speed $w_{||} = 0$. For non-dispersive or Alfvén waves this treatment of the phase speed is exact. Then the general transfer rates can easily be calculated without detailed knowledge of the particles' VDFs, which results in the simple approximations given in the previous sections, by which closure of the multi-fluid equations is achieved. This set of equations is supplemented by the evolution equation for the magnetic fluctuation energy, averaged over the range of wave vectors that are involved in the wave-particle interactions. Assuming a power-law for the wave spectrum, one can define a single growth/damping rate for the wave energy contained in the dissipation regime and a corresponding evolution equation.

If the complete physics of weak turbulence theory within the framework of QLT is to be retained, one needs to calcuate the full dispersion properties of the waves, which requires the knowledge of the two reduced VDFs defined in equations (13, 14). These evolve according to reduced diffusion equations (21, 22), which form a closed set if the plausible Gaussian approximation (26) is made. The reduced VDFs can be advanced in time solving the diffusion equation, which requires the knowledge of the transport coefficients. They require the actual dispersion properties, which in turn may change with the temperatures and bulk drifts of all species. In principle, all preparations to solve this closed loop of equations have been made in this paper. In practice, we believe that the simple closure schemes established here may provide a reasonable first step to describe the wave-particle interactions in a fluid-type picture, for which various levels of physical sophistication have been discussed.

The equations derived here may have various applications in the solar wind and corona in particular, where evidence (stated in the introduction) has been recently found for the possible role of cyclotron-resonant interactions between waves and protons or heavy ions. The observed excessive broadenings of EUV emission lines in the solar transition region and corona, especially in the direction perpendicular to the magnetic field, may indeed be indicative of wave heating of the kind discussed here. Our equations should be incorporated in future multi-fluid models of the solar corona and wind, improving the kinds of models put forward recently by Hu et al. (1997), Li et al. (1997), and Czechowski et al. (1998), or years ago in the solar wind context by Isenberg and Hollweg (1983).

In modelling the behaviour of minor ions in the solar wind, there has been a detailed debate (see e.g. Marsch et al., 1982a; Isenberg and Hollweg, 1983) about whether dispersive waves are able to produce sizable differential speeds and what the influence of dispersion is in this process. The general transport coefficients, if being put into a relaxation time form, clearly illustrate that it is the spectrum-averaged phase speed which determines the individual differential ion speed and heating. This species-specific phase speed may indeed differ substantially from the one obtained at $k = 0$, which is given by equation (43) that gives $V_p' \approx V_A$, since the fractional mass densities of all heavy ions are rather small. A detailed study will be carried out in the future to evaluate quantitatively the effects of wave dispersion and damping on the differential ion speed and temperature a given species might attain in the solar corona and wind.

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